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LETTER TO THE EDITOR

Comment on "Development of an Improved Gas-Kinetic BGK Scheme for Inviscid and Viscous Flows"

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Recently, Chae, Kim, and Rho proposed a new gas-kinetic BGK scheme [1]. In their approach, they modified the EFM or KFVS flux component in a gas-kinetic scheme through techniques based on Mach number splitting and Osher's linear subpath solution; see Eqs. (30) and (31) in [1]. In order to demonstrate the improvement in the numerical results obtained from their new scheme, in Figs. (12) and (14) they also included simulation results from another gas-kinetic BGK scheme, which was implicitly referred to Xu and Prendergast's BGK method [2]. In this letter, we point out the difference between the BGK scheme referred in paper [1] and Xu and Prendergast's BGK scheme [2]. Also, test cases similar to those presented in [1] will be calculated by Xu and Prendergast's BGK method.

The referred BGK scheme in the Chae-Kim-Rho paper is based on Eq. (19) for the flux evaluation,

$$\begin{aligned} f(0,0,t,u,v,\xi) = {} & \left(1-e^{-t/\tau}\right)g_0 + e^{-t/\tau} f_0(-ut,-vt) \\ & + \tau\left(-1+e^{-t/\tau}\right)(u\bar{a}+v\bar{b})g_0 \\ & + te^{-t/\tau}(u\bar{a}+v\bar{b})g_0, \end{aligned} \tag{0.1}$$

and the simulation results in Figs. (12) and (14) come from the above equation. If we compare the above equation with Eq. (3.5) in Xu and Prendergast's BGK scheme [2], we can see that Eq. (0.1) is different from Eq. (3.5). The gas distribution function (3.5) in [2] is the following (extended to the 2D case):

$$\begin{aligned} f(0,0,t,u,v,\xi) = {} & \left(1-e^{-t/\tau}\right)g_0 + e^{-t/\tau} f_0(-ut,-vt) \\ & + \tau\left(-1+e^{-t/\tau}\right)(u\bar{a}+v\bar{b}+A)g_0 \\ & + te^{-t/\tau}(u\bar{a}+v\bar{b})g_0 + tAg_0. \end{aligned} \tag{0.2}$$

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The main difference between Eq. (0.1) and Eq. (0.2) is that Eq. (0.1) ignores all terms related to A. In the following, we explain that Eq. (0.1) is inconsistent with the Chapman–Enskog expansion for the Navier–Stokes solution. From the gas-kinetic theory [3], the gas distribution function corresponding to the Navier–Stokes solution can generally be expressed as

$$f_{ns} = g + \phi,$$

where g is the equilibrium state and ϕ is the non-equilibrium part. The non-equilibrium part ϕ makes no contribution to the macroscopic conservative flow variables,

$$\int \phi \Psi d\Xi = 0,$$

where $\Psi = [1, u, v, \frac{1}{2}(u^2 + v^2 + \xi^2)]^T$ and $d\Xi = d\xi_1 d\xi_2 \cdots d\xi_K du dv$. Equation (0.1) does not correspond to the above Chapman–Enskog expansion for the viscous flow. This can be seen clearly in the smooth flow region. In this region, Eq. (0.1) goes to

$$g = g_0(1 - \tau(u\bar{a} + v\bar{b})).$$

In contrast, Eq. (0.2) gives

$$g = g_0(1 - \tau(u\bar{a} + v\bar{b} + A) + tA),$$

which has been proved and tested to be the Chapman–Enskog expansion for the viscous solution [4, 8]. The A term contributes to both the time evolution $g_0 tA$ and the physical viscous term $-\tau(u\bar{a} + v\bar{b} + A)g_0$. The conservation requirement for the non-equilibrium part is satisfied automatically, because

$$\int g_0(u\bar{a} + v\bar{b} + A)\Psi d\Xi = 0 \tag{0.3}$$

is exactly the equation to evaluate A in the smooth flow region. Therefore, in Xu and Prendergast's BGK method [2], the corresponding non-equilibrium part in the gas distribution function is $\phi = -\tau(u\bar{a} + v\bar{b} + A)g_0$, which is different from the non-equilibrium part $\phi = -\tau(u\bar{a} + v\bar{b})g_0$ in [1]. The non-equilibrium part in [1] cannot satisfy the requirement of $\int \phi \Psi d\Xi = 0$. Even for the steady state calculation, once there are spatial gradients in the flow variables, such that $\bar{a} \neq 0$ or $\bar{b} \neq 0$, the term A will not be zero (see Eq. (0.3)) and it must be kept for the viscous flow calculations.

After having the integral solution (0.2) of the BGK model, in order to test it numerically we need to define the particle collision time τ. As shown in [4], there are two parts in the collision time,

$$\tau = \frac{\mu}{p} + \Delta t \frac{p^l - p^\tau}{p^l + p^\tau}, \tag{0.4}$$

where μ is the viscosity coefficient, p is the pressure, and Δt is the time step. The first part

in the above equation corresponds to the physical viscosity and the second one gives the artificial dissipation in the case of flow discontinuities. With Xu and Prendergast's BGK distribution (0.2), we calculate the similar test cases, i.e., shock boundary layer interaction with $M = 2.0$ and $Re = 2.96 \times 10^5$, and laminar boundary layer with $Re = 10^4$ and $M = 0.2$. In both cases, the time step Δt is determined by the CFL condition with Courant number equal to 0.7. The non-slip boundary condition in both cases is implemented by generating two ghost cells inside the boundary with reversed fluid velocities with respect to the fluid above the boundary. Also, the van Leer limiter is used directly on all conservative variables for the construction of the initial slope inside each cell. All steady state solutions are obtained by the integration of the unsteady flow code. The codes of Xu and Prendergast's BGK scheme [1] for the above two cases can be obtained from the author's Web page (http://www.math.ust.hk/~makxu). The clustered rectangular mesh for the laminar boundary layer case and the simulation results for both cases are shown in Figs. 1–3. Since the mesh for Fig. (12) in [1] is generated according to the boundary layer solution, the boundary layer has the same number of grid points everywhere. In our calculation, the boundary layer has different numbers of grid points at different locations. As we can see from these figures, Xu and Prendergast's BGK scheme presents different results in comparison with Figs. (12) and (14) in [1]. During this work, it is also noticed that the cited experimental data in [1] for Fig. (14) seem different from the experimental data in [6, 7] even though they are assumed to come from the same experiment [5].

In conclusion, the BGK scheme in [1] is different from Xu and Prendergast's BGK method in [2]. In terms of the viscous boundary layer and shock boundary layer interaction cases, Xu and Prendergast's original scheme [2] and all other recent improvements [4] could do a very good job. In fact, the improvement necessary for the BGK scheme is the correct capture of the viscous shock structure. A nearly complete gas-kinetic BGK scheme for the correct capture of both boundary layer and shock structure has been proposed recently [9].

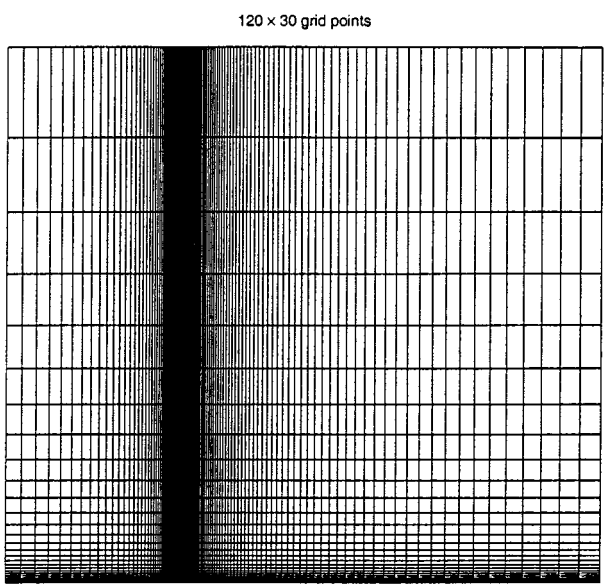

FIG. 1. 120×30 stretched rectangular mesh for the laminar boundary layer calculation. The mesh covers a computational domain $[-42, 120] \times [0, 130]$.

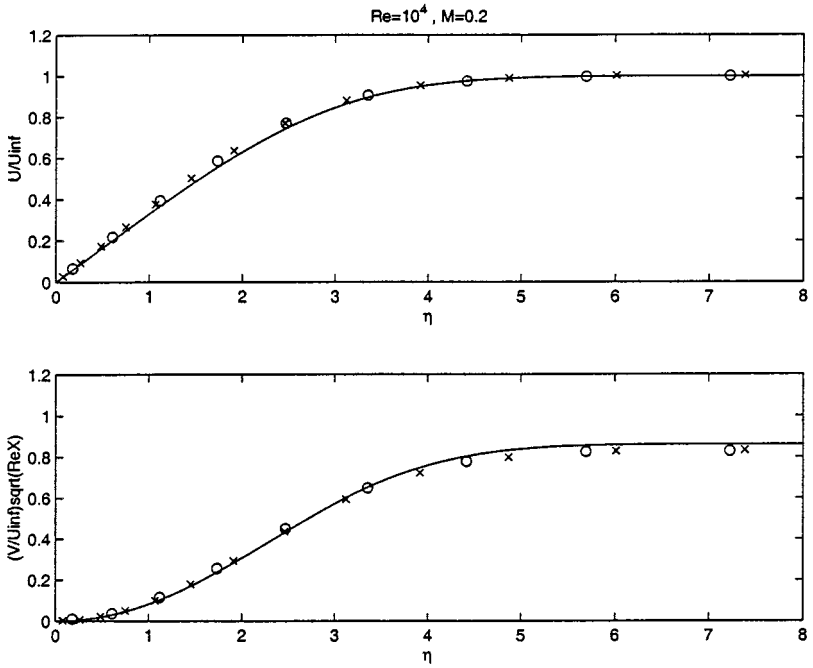

FIG. 2. U and V velocity distributions at the locations $x = 6.438$ (circles) and $x = 34.469$ (cross sign). Both solid lines are the exact solutions. Free stream conditions are $M = 0.2$ and $Re = 10^4$.

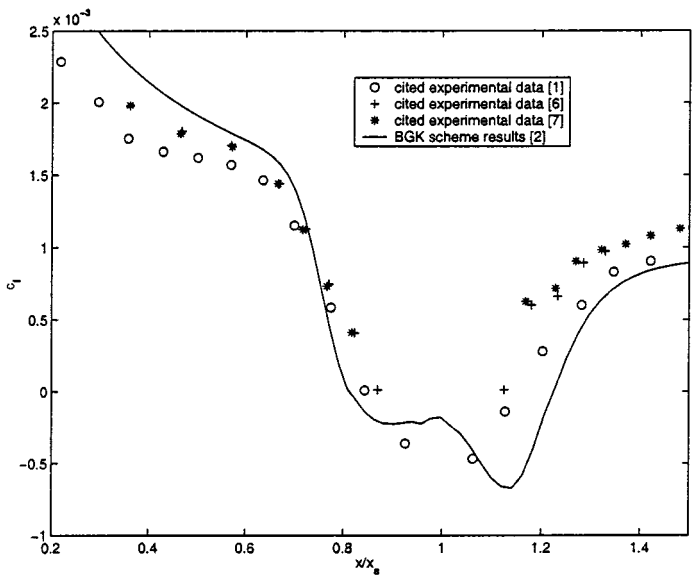

FIG. 3. Shock boundary layer interaction case at $M = 2.0$ and $Re = 2.96 \times 10^5$ on a mesh of 106×66 grid points. Skin friction coefficient c_f along the flat plate is compared with the experimental data.

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